Question 18

If vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are orthogonal, what is the component of $\vec{\mathbf{B}}$ along the direction of $\vec{\mathbf{A}}$?
What is the component of $\vec{\mathbf{A}}$ along the direction of $\vec{\mathbf{B}}$?

Solution

Suppose that $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are orthogonal. Then their dot product is zero.

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = 0$$

The component of $\vec{\mathbf{B}}$ along the direction of $\vec{\mathbf{A}}$ is the dot product of $\vec{\mathbf{B}}$ with a unit vector in the direction of $\vec{\mathbf{A}}$:

$$\vec{\mathbf{B}} \cdot \frac{\vec{\mathbf{A}}}{\left|\vec{\mathbf{A}}\right|} = \frac{\vec{\mathbf{B}} \cdot \vec{\mathbf{A}}}{\left|\vec{\mathbf{A}}\right|}$$

$$= \frac{\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}}{\left|\vec{\mathbf{A}}\right|}$$

$$= \frac{0}{\left|\vec{\mathbf{A}}\right|}$$

$$= 0.$$

The component of $\vec{\mathbf{A}}$ along the direction of $\vec{\mathbf{B}}$ is the dot product of $\vec{\mathbf{A}}$ with a unit vector in the direction of $\vec{\mathbf{B}}$:

$$\vec{\mathbf{A}} \cdot \frac{\vec{\mathbf{B}}}{\left|\vec{\mathbf{B}}\right|} = \frac{\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}}{\left|\vec{\mathbf{B}}\right|}$$

$$= \frac{0}{\left|\vec{\mathbf{B}}\right|}$$

$$= 0.$$